Poisson Statistics of Radioactive Decay

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Poisson statistics were studied using the radioactive decay of ^{137}Cs as a source. A scintillation counter measured gamma rays emitted by ^{137}Cs as well as background from cosmic rays and other experiments in the laboratory. Data from approximate mean rates of 1, 4, 10 and 100 counts/sec was compared to both theoretical Poisson distributions and Monte Carlo simulations. The reduced chi squared values for the fits to theoretical distributions ranged from 0.57 to 2.13. The values from the Monte Carlo simulations were within the margin of statistical error of the data. Thus, it was found that gamma rays from the radioactive decay of ^{137}Cs with added background events is accurately modeled by Poisson statistics, so each decay event in a bulk source is a random, independent event.

1. INTRODUCTION

Poisson statistics describe random, independent events that occur at a fixed mean rate μ. Each decay event in a bulk source of ^{137}Cs is random, independent and occurs at a fixed mean rate μ, so we can model this radioactive decay using Poisson statistics. The Poisson distribution can be considered to be a limiting case of the binomial distribution, and the Gaussian distribution can be considered to be a limiting case of the Poisson distribution. We review the theoretical basis for these three important probability distributions, then compare our experimental results to both theoretical and simulated results.

2. THEORY

2.1. The Binomial Distribution

The binomial distribution(2) describes the probability $P_B(x)$ describes the probability of observing x of n total items to be in a certain state with probability p. There are number of ways to choose x of n items is,

$$\binom{n}{x} = \frac{n!}{x!(n-x)!} \tag{1}$$

The x events occur with probability p each, and the $(n-x)$ events occur with probability $(1-p)$ each, so with we have the binomial distribution,

$$P_B(x) = \frac{n!}{x!(n-x)!}p^x(1-p)^{n-x} \tag{2}$$

[1]

2.2. The Poisson Distribution

The Poisson distribution can be derived from the binomial distribution by letting $n \to \infty$ and fixing the mean rate $\mu = np$. Equivalently, we can let $p \to 0$. Then there is no longer a fixed number of possible events, so each new event does not depend on the number of evens that has already occured, so the events are statistically independent. In the limit, we have,

$$\frac{n!}{(n-x)!} = n(n-1)\cdots(n-x-2)(n-x-1) = n^x \tag{3}$$

$$(1-p)^{n-x} = (1-p)^{-x}(1-p)^n = (1+px)^{-x}(1-p)^{\frac{\mu}{p}} = e^{-\mu} \tag{4}$$

$$P_p(x;\mu) = \frac{1}{x!}\frac{n!}{(n-x)!}p^x(1-p)^{-x} = \frac{\mu^x}{x!}e^{-\mu} \tag{5}$$

[1]

2.3. The Gaussian Distribution

It is interesting to note that in the for a large mean counting rate μ, the Poisson distribution approaches the Gaussian distribution,

$$P_G(x;\mu) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{1}{2}(\frac{x-\mu}{\sigma})^2\right) \tag{6}$$

[1]

Figure 1 shows a comparison between the theoretical Poisson distribution and the theoretical Gaussian distribution for $\mu = 4$ counts/sec. Even here, we see a good agreement between the two distributions, with a reduced chi squared value of 0.19. As μ increases, the agreement becomes better.

3. EXPERIMENT

a. Figure 2 shows a schematic of the experimental setup. A bulk sample of ^{137}Cs was used as a gamma ray

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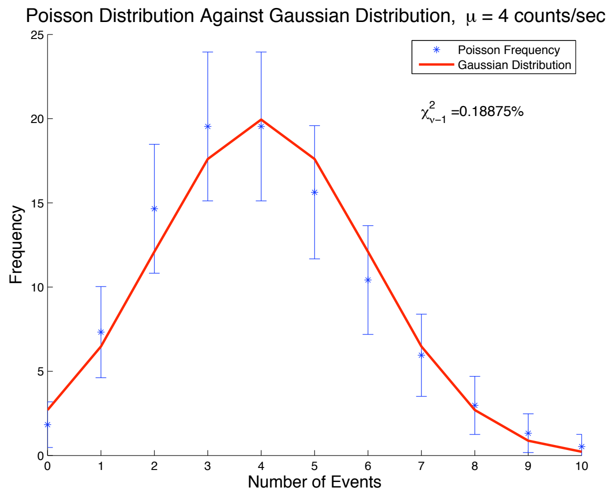

FIG. 1: A comparison of the Poisson distribution and the Gaussian distribution for $\mu = 4$ counts/sec. Even at this low μ value, the Gaussian distribution falls within statistical error of the Poisson distribution. As μ was increased, the two distributions were found to quickly converge.

source. ^{137}Cs emits a gamma particle when it undergoes radioactive decay. In a large sample, each gamma emission is a random, statistically independent event, and emissions occur at a fixed mean rate. Other background gamma rays included cosmic rays and gamma rays from other experiments in the laboratory. We assume that these background events also occur at a fixed mean rate.

b. The gamma rays then passed through an NaI scintillator. In a scintillator, photons excite the crystal and lose some energy in the process, then the excited crystal emits photons. The photons from the scintillator then traveled to the photomultiplier tube. In a photomultiplier tube, photons first strike the photocathode at the end and eject electrons due to the photoelectric effect. The high voltage power supply maintains a large potential difference between the two ends of the photomultiplier tube. Dynodes, a kind of electrode, are arranged down the tube with increasing voltage. Electrons travel down the tube and gain energy, then striking a dynode and releasing more electrons. This causes an avalanche of electrons, and finally an electrical pulse at the photomultiplier tube output. Typical gains of such dynode chains range from several thousand to one million.[2]

c. The electrical pulse from the photomultiplier was passed through a preamplifier and then an amplifier, which both amplified the signal. The amplified signal was then passed through a discriminator, which rejected all pulses below a certain threshold voltage. Finally, the signal from the discriminator was passed to the counter. The counter counted how many pulses it received in adjustable, fixed time intervals.

d. We expected the pulse output from the preamplifier and the pulse output from the amplifier to be in one to one correspondence, but instead we found that the number of signals coming out of the amplifier was orders of magnitude greater than the number of signals coming out of the preamplifier. Possible causes of this discrepancy were that the the amplifier was picking up signals that did were not triggered on the oscilliscope we were using to view these pulses, and that the amplifier was picking up noise from the preamplifier. However, as long as the pulses coming out of the amplifier were statistically independent and occured at a fixed mean rate, any discrepancy would not affect our experimental results. Another issue was that the timer on our counter was calibrated incorrectly. When the timer counted 1 second, approximately 4 seconds actually passed. So, when we wanted the counter to count the number of events in 1 second, we actually adjusted the time to 0.6 seconds to achieve approximate 1 second invervals. For the rest of this paper, let “1 second” mean “0.6 seconds as defined by our faulty counter.” A real second and our approximate second can be used interchangeably, because as long as our approximate second was consistent, it does not affect our Poisson statistics analysis.

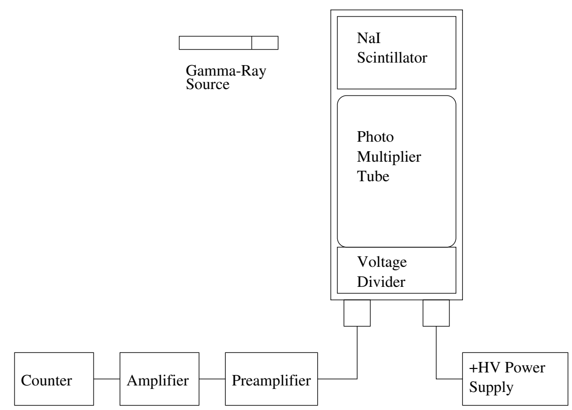

FIG. 2: Schematic of the experimental setup. Adapted from lab guide[3].

4. DATA AND ANALYSIS

e. We measured the number of counts that occured in 1 second 100 times for fixed mean rates μ of approximately 1, 4, 10 and 100 counts per second. The actual fixed mean rates μ we achieved were 0.98, 3.69, 8.02 and 102.92 counts/sec. We also measured the count numbers for 4 long 100 second trials, one for each of the mean counting rates. We set the timer differently for the short and long trials so that the time of the long 100 second trial was not equal to the time of 100 of the 1 second trials. Therefore, we could make no comparison of the two, so the long trials are not included in this analysis.

f. The main source of error in all of our measurements was assumed to be statistical and to follow Poisson statistics, so the error on a single measured number x was always assumed to be $\sqrt{x}$. When the final reported value was a function of many measurements x_i each with error $\sqrt{x_i}$, we used error propogation techniques to find the error on the calculated value.[1]

g. First, we checked that our events occured at a fixed mean rate by plotting the cumulative average of μ as a function of the count number. Figure 3 shows this plot for $\mu \approx 4$. For $\mu \approx 1$, 4, 10 and 100, we saw the average μ converge to a value, so we can conclude that the mean count rate did not fluctuate over the time scales we were considering. These cumulative averages with errors are listed in table I.

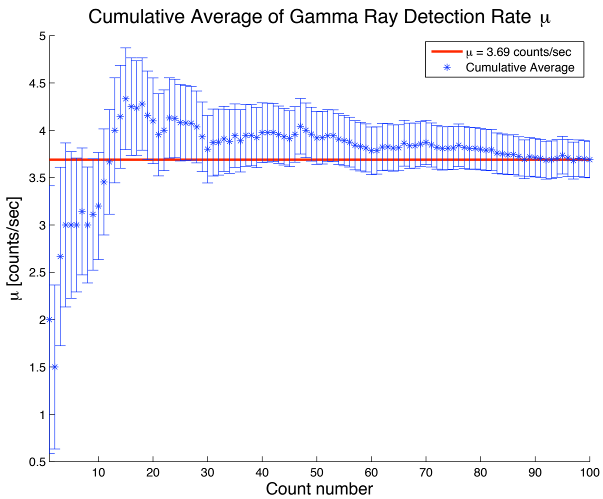

FIG. 3: Cumulative average of the gamma ray detection rate μ for $\mu \approx 4$ counts/sec. μ varies between 1.50 and 4.33 counts/sec, and the series of average values eventually converges around $\mu = 3.69$ counts/sec. The plots of the cumulative average for $\mu \approx 1$, 10 and 100 counts/sec behave similarly.

h. Then we checked the frequency distributions of our data for $\mu \approx 1$, 4, 10 and 100 against theoretical Poisson distributions for our measured average μ of 0.98, 3.69, 8.02 and 102.92 counts/sec. The theoretical frequency distributions were generated by multiplying the Poisson probability distribution (5) by 100, for the 100 data points. The plot for $\mu = 8.02$ counts/sec is shown in figure 4, and the plots for the other μ are similar. The reduced chi squared values of our theoretical fits to our measured data are $\chi^2_{\nu-1} = 0.94$, 1,20, and 0.57 for $\mu = 0.98$, 3.69, 8.02 and 102.92 counts/sec, respectively.

i. In addition to our theoretical fits, we also ran Monte Carlo simulations in Matlab that generated Poisson distributions for our input μ that matched our calculated average μ of 0.98, 3.69, 8.02 and 102.92 counts/sec. Like in our experiment, we took the counts simulated in 100 1 second intervals, and made frequency plots. A plot of a typical Monte Carlo output for $\mu = 8.02$ is shown in figure 5. For a given μ, we ran the simulation 10 times and calculated 10 different μ and 10 different σ, the standard deviation of each set of 100 values. We then calculated our errors on those simulated values by finding the standard deviations of the set of 10 values. A comparison between experimental and simulated μ and σ is shown in table (I).

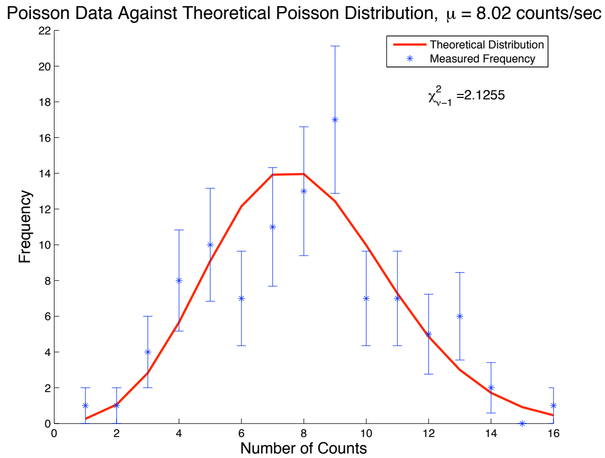

FIG. 4: Measured frequency distribution with a calculated $\mu = 3.69 \pm .19$ counts/sec plotted against a theoretical poisson distribution with $\mu = 3.69$ counts/sec. The theoretical and experimental distributions agree well, with a reduced chi squared value $\chi_{\nu-1}$ of 2.13. Comparisons were also done for $\mu = 0.98, 8.02$ and 102.92 counts/sec with $\chi_{\nu-1}$ of 0.94, 1.20, and .57, respectively.

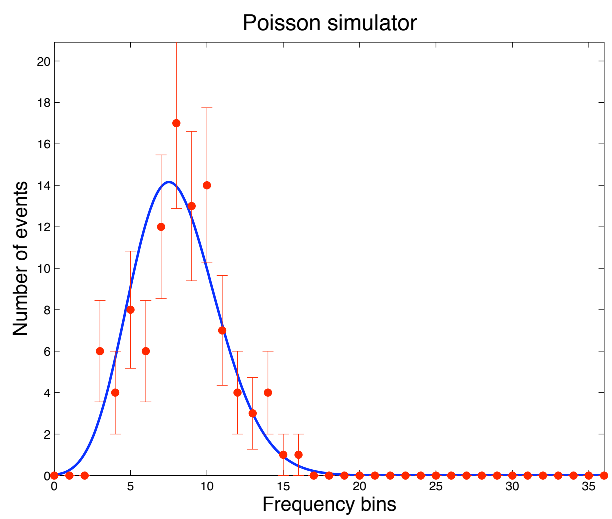

FIG. 5: Monte Carlo-generated Poisson distribution with input $\mu = 8.02$ counts/sec. Output from 8.13 Matlab script.

5. CONCLUSIONS

The fits of our data to theoretical Poisson distributions have $\chi^2_{\nu-1}$ values of 0.94, 1.20, 2.13, and 0.57 for $\mu = 0.98$, 3.69, 8.02 and 102.92 counts/sec, respectively, indicating a good fit for every value of μ. In addition,

	$\mu \approx 1$/s	$\mu \approx 4$/s	$\mu \approx 10$/s	$\mu \approx 100$/s
$\overline{\mu}_{\text{meas}}$	0.98 ± 0.10	3.69 ± 0.19	8.02 ± 0.28	102.92 ± 1.01
$\overline{\mu}_{\text{sim}}$	0.94 ± 0.11	3.68 ± 0.11	7.95 ± 0.34	102.62 ± 0.67
σ_{meas}	$1.12 \pm .23$	$1.69 \pm .17$	3.04 ± 0.21	10.07 ± 0.20
σ_{sim}	0.98 ± 0.083	1.90 ± 0.11	2.79 ± 0.18	10.17 ± 0.78

TABLE I: Measured mean rate $\overline{\mu}_{\text{meas}}$, simulated mean rate $\overline{\mu}_{\text{sim}}$, measured standard deviation σ_{meas}, and simulated standard deviation σ_{sim} of each of the four 100-trial distributions.

simulated data using Monte Carlo methods and an assumed Poisson distribution is within the statistical error of our experimental data. Therefore, we can conclude that the radioactive decay of ^{137}Cs is a Poisson process. That is, radioactive decay events in a bulk sample of ^{137}Cs are statistically independent and occur at a fixed mean rate. We can also conclude that gamma rays from background sources in the laboratory were Poisson.

[1] P. Bevington and D. Robinson, *Data Reduction and Error Analysis for the Physical Sciences* (McGraw-Hill, 2003).
[2] A. Melissinos and J. Napolitano, *Experiments in Modern Physics* (Academic Press, 2003).
[3] J. L. Staff, *Poisson Statistics* (2007).

Acknowledgments

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